Southern York County School District Instructional Plan

Course/Subject: AP Calculus AB Grade Level: 11,12	
Textbook(s)/Instructional Materials Used: Calculus (8th Edition); ISBN # 0-618-50298-X; Houghton Mifflin	
Dates: Full Year	
Stage 1 – Desired Results	
College Board AP Calculus AB/BC Curriculum Framework Addressed: **This unit is designed to review all necessary Precalculus Essential Understandings, Learning Objectives, and Essential Knowledge as published in the preceding Precalculus Unit Plan.** **CC.2.3.HS.A.14 - Apply geometric concepts to model and solve real world problems.** **CC.2.2.HS.C.1 - Use the concept and notation of functions to interpret and apply them in terms of their context.** **CC.2.2.HS.C.2 - Graph and analyze functions and use their properties to make connections between the different representations.** **CC.2.2.HS.C.3 - Write functions or sequences that model relationships between two quantities.** **CC.2.2.HS.C.4 - Interpret the effects transformations have on functions and find the inverses of functions.** **CC.2.2.HS.C.5 - Construct and compare linear, quadratic, and exponential models to solve problems.** **CC.2.2.HS.C.6 - Interpret functions in terms of the situations they model.** **CC.2.2.HS.C.7 - Apply radian measure of angle and unit circle to analyze the trigonometric functions.** **CC.2.2.HS.D.1 - Interpret the structure of expressions to represent a quantity in terms of its context.** **CC.2.2.HS.D.4 - Understand the relationship between zeros and factors of polynomials to make generalizations about functions and their graphs.** **CC.2.2.HS.D.7 - Create and graph equations or inequalities to describe numbers or relationships.** **CC.2.2.HS.D.9 - Use reasoning to solve equations and justify the solutions method.** **CC.2.1.HS.F.1 - Apply and extend the properties of exponents to solve problems with rational exponents.** **CC.2.1.HS.F.4 - Use units as a way to understand problems and to guide the solution of multi-step problems.** **CC.2.1.HS.F.5 - Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.**	
Understanding(s): ***Students will understand . . .*** 1. Parent graphs and their transformations; provide a connection between their graphs and equations (CC.2.2.HS.C.1, CC.2.2.HS.C.4, CC.2.2.HS.C.5, CC.2.1.HS.F.4) 2. Data isn't always perfect so we must be able to interpret models for real-world situations (CC.2.2.HS.D.1, CC.2.1.HS.F.5, CC.2.1.HS.F.4) 3. Ways in which functions are very useful tools in mathematics to model the real world. (CC.2.2.HS.C.2, CC.2.2.HS.C.6, CC.2.3.HS.A.14, CC.2.2.HS.C.5, CC.2.2.HS.D.9, CC.2.1.HS.F.4)	**Essential Question(s):** • What skills must I possess as a student to succeed in AP Calculus AB? • What specific areas do I need review in order to move forward? • What precalculus skills do I need to have mastered in order to succeed on the AP Calculus AB Exam? • How is what I learned last year going to be connected to what I will learn this year?
Learning Objectives:	

Students will know . . .	Students will be able to:
• Symmetry and its relationship to functions • Intersection points, zeros, and their applications and connections (CC.2.2.HS.D.9, CC.2.2.HS.D.4) • Transcendental functions including their manipulation and application (CC.2.2.HS.C.2) • Mathematical models and the purpose they serve (CC.2.2.HS.C.6, CC.2.3.HS.A.14, CC.2.1.HS.F.5) • The formal and symbolic definitions of domain (CC.2.2.HS.C.2) • The formal and symbolic definitions of range (CC.2.2.HS.C.2)	• Apply their basic mathematical knowledge to more involved and complex problems. (CC.2.2.HS.D.9) • Graph complex equations by hand and using a graphing technology (CC.2.2.HS.D.7) • Test an equation or graph for symmetry (CC.2.2.HS.C.2) • Mathematically model data by hand and using graphing technology (CC.2.2.HS.C.6) • Find the domain and range of a function (CC.2.2.HS.C.2) • Determine if a graph or equation is a function (CC.2.2.HS.C.2) • Find the composition of two functions (CC.2.2.HS.C.2) • Determine if a function is even or odd (CC.2.2.HS.C.2) • Apply their functional understanding to trigonometric functions (CC.2.2.HS.C.7) • Apply properties of rational exponents (CC.2.1.HS.F.1)

Dates: September	**Unit Plan: Limits**
Stage 1 – Desired Results	
College Board AP Calculus AB/BC Curriculum Framework Addressed: **All Essential Understandings, Learning Objectives, and Essential Knowledge cited below are taken from the** ***AP Calculus AB and AP Calculus BC Course and Exam Description (Effective Fall 2016)*** **The idea of limits is essential for the development of calculus ideas and concepts as many major aspects of the content covered are first developed from a discrete model. A proper, intuitive comprehension of limits and their properties are essential to furthering this understanding later in the course. This knowledge base includes but is not limited to one-sided limits, limits at infinity, the limit of a sequence, and infinite limits. Furthermore, students must be able to work with these concepts using multiple representations including tables, graphs, as well as a given function or relation. Finally, students must be able to apply the ideas of limits to determine continuity of a given function.**	
Understanding(s):	**Essential Question(s):**
Students will understand . . . • The concept of a limit can be used to understand behavior of functions. (EU 1.1) • Continuity is a key property of functions that is defined using limits. (EU 1.2)	• Why is the study of limits important? • How can limits be used and applied to familiar situations we witness in everyday life? • In what professions, fields, or actual situations might our applications of limits in this chapter be useful?
Learning Objectives:	

Students will know . . .	*Students will be able to:*
• Given a function f, the limit of f(x) as x approaches c is a real number R if f(x) can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x \to c} f(x) = R$. (EK 1.1A1) • The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits. (EK 1.1A2) • A limit might not exist for some functions at particular values of x. Some ways that the limit might not exist are if the function is unbounded, if the functions i oscillating near this value, or if the limit from the left does not equal the limit from the right. (EK 1.1A3) • Numerical and graphical information can be used to estimate limits. (EK 1.1B1) • Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules. (EK 1.1C1) • The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem. (EK 1.1C2). • Limits of the indeterminate forms 0/0 and ∞/∞ may be evaluated using L'Hopital's Rule. (EK 1.1C3) • Asymptotic and unbounded behavior of functions can be explained and described using limits. (EK 1.1D1) • Relative magnitudes of functions and their rates of change can be compared using limits. (EK 1.1D2) • A functoin f is continuous at x=c provided that f(c) exists, $\lim_{x \to c} f(x)$ exists, and $\lim_{x \to c} f(x) = f(c)$ (EK 1.2A1) • Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains. (EK 1.2A2). • Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes. (EK 1.2A3). • Continuity is an essential condition for theorems such as the Intermediate Value	• Express limits symbolically using correct notation. (LO 1.1A(a)) • Interpret limits expressed symbolically. (LO 1.1A(b)). • Estimate limits of functions (LO 1.1B) • Determine limits of functions. (LO 1.1C) • Deduce and interpret behavior of functions using limits (LO 1.1D) • Analyze functions for intervals of continuity or points of discontinuity (LO 1.2A) • Determine applicability of important calculus theorems using continuity. (LO 1.2B)

Theorem, the Extreme Value Theorem, and the Mean Value Theorem. (EK 1.2B1)	
Dates: October, November, Decemeber	**Unit Plan: Derivatives and Applications**

Stage 1 – Desired Results

College Board AP Calculus AB/BC Curriculum Framework Addressed: **All Essential Understandings, Learning Objectives, and Essential Knowledge cited below are taken from the *AP Calculus AB and AP Calculus BC Course and Exam Description (Effective Fall 2016)*** **Derivatives are essentially used to describe the rate of change of one variable with respect to another. As such, they are essential to a calculus course which is, at its most basic level, the mathematical study of movement. Applications of derivatives including but not limited to calculating the slope of a tangent line both generally and at a point, analyzing the graph of a function, and solving problems involving rectilinear motion. Just like limits, students should be able to apply their understanding across multiple representations. Finally, students must be able to extend their knowledge base to skills such as working with differential equations, applying the Mean Value Theorem, and real-world applications such as related rates, optimization, and growth and decay models.**	
Understanding(s): ***Students will understand . . .*** ● The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies(EU 2.1) ● A function's derivative, which is itself a function, can be used to understand the behavior of a function (EU 2.2) ● The derivative has multiple interpretations and applications including those that involves instantaneous rates of change (EU 2.3) ● The Mean Value Theorem connects the behavior of a differentiable function over an interval to the behavior of the derivative of that function at a particular point in the interval (EU 2.4)	**Essential Question(s):** ● Why is the study of derivatives important? ● How do derivatives relate to limits and, projecting forward, how will they relate to integrals? ● What are derivatives and how can they be applied in the physical world through the lense of upper -level mathematics? ● In what professions, fields, or actual situations might our use of derivatives in this unit be useful?
Learning Objectives: ● ***Students will know . . .*** ● The difference quotients $\frac{f(a+h)-f(a)}{h}$ and $\frac{f(x)-f(a)}{x-a}$ express the average rate of change of a function over an interval. (EK 2.1A1) ● The instantaneous rate of change of a fucnion at a point can be expressed by $\lim_{x \to 0} \frac{f(a+h)-f(a)}{h}$ or $\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$, provided the limit exists. These are common forms of the definition of the derivative and are denoted f'(a). (EK 2.1A2) ● The derivative of f is the fucntion whose value at x is $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ provided this limit exists. (EK 2.1A3)	● ***Students will be able to:*** ● Identify the derivative of a function as the limit of a difference quotient (LO 2.1A) ● Estimate derivatives (LO 2.1B) ● Calculate derivatives (LO 2.1C) ● Determine higher order derivatives (LO 2.1D) ● Use derivatives to analyze properties of a function (2.2A) ● Recognize the connection between differentiability and continuity (LO 2.2B) ● Interpret the meaning of a derivative within a problem (LO 2.3A)

- For y=f(x), notations for the derivative include dy/dx, f’(x), and y’. (EK 2.1A4)
- The derivative can be represented graphically, numerically, analytically, and verbally. (EK 2.1A5)
- The derivative at a point can be estimated from information given in tables or graphs (EK 2.1B1)
- Direct application of the definition of the derivative can be sued to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions. (EK 2.1C1)
- Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse functions. (EK 2.1C2)
- Sums differences, products, and quotients of functions can be differentiated using derivative rules. (EK 2.1C3)
- The chain rule provides a way to differentiate composite functions. (EK 2.1C4)
- The chain rule is the basis for implicit differentiation. (EK 2.1C5)
- The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists. (EK 2.1C6)
- Differentiating f’ produces the second derivative f”, provided the derivative of f’ exists; repeating this process produces higher order derivatives of f. (EK 2.1D1)
- Higher order derivatives are represented with a variety of notations. For y=f(x), notations for the second derivative include $\frac{d^2y}{dx^2}$, f”(x), and y”. Higher order derivatives can be denoted $\frac{d^ny}{dx^n}$ or $f^{(n)}(x)$. (EK 2.1D2)
- First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection. (EK2.2A1)
- Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations. (EK 2.2A2)

- Solve problems involving the slope of a tangent line (LO 2.3B)
- Solve problems involving related rates, optimization, rectilinear motion (LO 2.3C)
- Solve problems involving rates of change in applied contexts (LO 2.3D)
- Verify solutions to differential equations (LO 2.3E)
- Estimate solutions to differential equations (LO 2.3F)
- Apply the Mean Value Theorem to describe the behavior of a function over an interval. (LO 2.4A)

• Key features of the graphs of f, f', and f" are related to one another. (EK 2.2A3) • A continuous function may fail to be differentiable at a point in its domain. (EK 2.2B1) • If a function is differentiable at a point, then it is continuous at that point. (EK 2.2B2) • The unit for f'(x) is the unit for f divided by the unit for x. (EK 2.3A1) • The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable. (EK 2.3A2) • The derivative at a point is the slope ofthe line tangent to a graph at that point on the graph. (EK 2.3B1) • The tangent line is the graph of a locally linear approximation of the function near the point of tangency. (EK 2.3B2) • The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration. (EK 2.3 C1) • The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known. (EK 2.3C2) • The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval. (EK 2.3C3) • The derivative can be used to express information about rates of change in applied contexts. (EK 2.3D1) • Solutions to differential equations are functions or families of functions (EK 2.3E1) • Derivatives can be used to verify that a function is a solution to a given differential equation. (EK 2.3E2) • Slope fields provide visual clues to the behavior of solutions to first order differential equations. (EK 2.3F1) • If a function f is continuous over the interval [a,b] and differentiable over the interval (a,b) the Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval. (EK 2.4A)	

Dates: January, February, March	Unit Plan: Integrals and Applications
Stage 1 – Desired Results	
College Board AP Calculus AB/BC Curriculum Framework Addressed: **All Essential Understandings, Learning Objectives, and Essential Knowledge cited below are taken from the** ***AP Calculus AB and AP Calculus BC Course and Exam Description (Effective Fall 2016)*** **While at their most basic level, integrals can be thought of as the antiderivative, they have a number of applications critical to the study of calculus concepts. These applications begin with the approximation of a definite integral in the form of a Riemann sum and continue to the computation of definite integrals using basic methods, u-substitution, and later integral by parts and integration using partial fractions. Similar to derivatives, students must be able to apply their understanding the rectilinear motion applications as well as understand that a definite integral can be used as an accumulation function. Finally, students must be able to conceptualize the relationship between differentiation and integration which is formalized in the Fundamental Theorem of Calculus.**	
Understanding(s): ***Students will understand . . .*** 1. Antidifferentiation is the inverse process of differentiation. (EU 3.1) 2. The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies. (EU 3.2) 3. The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration. (EU 3.3) 4. The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation. (EU 3.4) 5. Antidifferentiation is an underlying concept involved in solving separable differential equations. Solving separable differential equations involves determining a function or relation given its rate of change. (EU 3.5)	**Essential Question(s):** ● Why is the study of integrals important? ● How do integrals relate to limits and derivatives? ● What are integrals and how can they be applied in the physical world through the lense of upper -level mathematics? ● In what professions, fields, or actual situations might our use of integrals in this unit be useful?
Learning Objectives: ***Students will know . . .*** ● An antiderivative of a function f is a function g whose derivative is f. (EK 3.1A1) ● Differentiation rules provide the foundation for finding antiderivatives. (EK 3.1A2) ● A Riemann sum, which requires a partition of an interval, I, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of the subinterval of the partition. (EK 3.2A1)	***Students will be able to:*** ● Recognize antiderivatives of basic functions. (LO 3.1A) ● Interpret the definite integral as the limit of a Riemann sum. (LO 3.2A(a)) ● Express the limit of a Riemann sum in integral notation (LO 3.2A(b)) ● Approximate a definite integral. (LO 3.2B) ● Calculate a definite integral using areas and properties of definite integrals. (LO 3.2C)

- The definite integral of a continuous function f over the interval [a,b], denoted by $\int_a^b f(x)dx$, is the limit of a Riemann sums as the widths of the subintervals approach 0. That is, $\int_a^b f(x)dx = \lim_{max\Delta x_i \to 0} \sum_{i=1}^{n} f(x_i)\Delta x_i$ where x_i is a value in the *i*th subinterval, Δx_i is the width of the *i*th subinterval, n is the number of subintervals, and $max\Delta x_i$ is the width of the largest subinterval. Another form of the definition is $\int_a^b f(x)dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$ where $\Delta x_i = \frac{b-a}{n}$ and x_i is a value in the *i*th subinterval. (EK 3.2A2)
- The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral. (EK 3.2A3)
- Definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, and verbally (EK 3.2B1)
- Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions. (EK 3.2B2)
- In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area. (EK 3.2C1)
- Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals. (EK 3.2C2)
- The definition of the definite integral may be extended to functions with removable or jump discontinuities. (EK 3.2C3).
- The definite integral can be used to define new functions, for example, $f(x) = \int_0^x e^{-t^2} dt$. (EK 3.3A1)

- Analyze functions defined by an integral (LO 3.3A)
- Calculate antiderivatives. (LO 3.3B(a))
- Evaluate definite integrals. (LO 3.3B(b))
- Interpret the meaning of a definite integral within a problem. (LO 3.4A)
- Apply definite integrals to problems involving the average value of a function (LO 3.4B)
- Apply definite integrals to problems involving motion. (LO 3.4C)
- Apply definite integrals to problems involving area and volume. (LO 3.5D)
- Use the definite integral to solve problems in various contexts (LO 3.4E)
- Analyze differential equations to obtain general and specific solutions. (LO 3.5A)
- Interpret, create, and solve differential equations from problems in context. (LO 3.5B)

- If f is a continuous function on the interval [a,b], then $d/dx\left(\int_a^x f(t)dt\right) = f(x)$, where x is between a and b. (EK 3.3A2)
- Graphical, numerical, analytical, and verbal representations of a function f provide information about the function g defined as $g(x) = \int_a^x f(t)dt$. (EK 3.3A3)
- The function defined by $F(x) = \int_a^x f(t)dt$ is an antiderivative of f. (EK 3.3B1)
- If f is continuous on the interval [a,b] and F is an antiderivative of f, then $\int_a^b f(t)dt = F(b) - F(a)$. (EK 3.3B2)
- The notation $\int f(x)dx = F(x) + C$ means that $F'(x) = f(x)$ and $\int f(x)dx$ is called an indefinite integral of the function f. (EK 3.3B3)
- Many functions do not have closed form antiderivatives. (EK 3.3B4)
- Techniques for finding antiderivatives include algebraic manipulation, such as long division and completing the square and substitution of variables. (EK 3.3B5)
- A function definied as an integral represents an accumulation of a rate of change. (EK 3.4A1)
- The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval. (EK 3.4A2)
- Teh limit of an approximating Riemann sum can be interpreted as a definite integral. (EK 3.4A3)
- The average value of a function f over an interval [a,b] is $\frac{1}{b-a}\int_a^b f(x)dx$. (EK 3.4B1)
- For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time and the definite integral of speed represents the particle's total distance traveled over the interval of time. (EK 3.4C1)
- Areas of certain regions in the plane can be calculated with definite integrals. (EK 3.4D1)

- Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals. (EK 3.4D2) - The definite integral can be used to express information about accumulation and net change in many applied contexts. (EK 3.4E1) - Antidifferentiation can be used to find specific solutions to differential equations with given initial conditions, including applications to motion along a line as well as exponential growth and decay. (EK 3.5A1) - Some differential equations can be solved by separation of variables. (EK 3.5A2) - Solutions to differential equations may be subject to domain restrctions. (EK 3.5A3) - The function F defined by $F(x) = c + \int_a^x f(t)dt$ is a general solution to the differential equation $dy/dx = f(x)$, and $F(x) = y_0 + \int_a^x f(t)dt$ is a particular soltuion to the differential equation $dy/dx = f(x)$ satisfying $F(a) = y_0$. (EK 3.5A4) - The model for exponential growth and decay that arises from the statement “The rate of change of a quantity is proportional to the size of the quantity” is $dy/dt = ky$. (KEK 3.5B1)	

Dates: April/May	**Unit Plan: Application of Calculus Concepts and AP Exam Review**
Stage 1 – Desired Results	
College Board AP Calculus AB/BC Curriculum Framework Addressed: **This unit is designed to review all AP Calculus AB & BC Essential Understandings, Learning Objectives, and Essential Knowledge as published in the *AP Calculus AB and BC Course and Exam Description (Effective Fall 2016). This unit is designed to review course content for the AP Exam.***	
Understanding(s):	**Essential Question(s):**

Students will understand . . . 1. Each and every concept covered throughout the AP Calculus AB course. This includes but is not limited to content related to: a. Limits b. Derivatives c. Integrals 2. Ways in which calculus is useful in mathematics 3. Ways in which calculus is useful in real-world settings outside of the classroom	● What concepts and skills do I need to review in order to be fully prepared for the AP Calculus Exam? ● What are my strengths and weaknesses in regard to the content that will be on the AP Calculus Exam? ● How are each of the major areas, specifically limits, derivatives, and integrals related and how will they be presented on the AP Calculus Exam?
Learning Objectives: ***Students will know . . .*** ● This particular unit is unique as it is a cumulative review of the entire AP Calculus AB course. All major concepts and ideas covered will be revisited and reviewed.	***Students will be able to:*** ● This particular unit is a cumulative review of the entire AP Calculus AB course. Each skill the students have been exposed to this last year will be reviewed through practice and assessment.