Southern York County School District Instructional Plan

Course/Subject: AP Calculus BC Grade Level: 12
Textbook(s)/Instructional Materials Used: Calculus (8th Edition); ISBN # 0-618-50298-X; Houghton Mifflin
Dates: Full Year
Stage 1 – Desired Results
College Board AP Calculus AB/BC Curriculum Framework Addressed: **This unit is designed to review all AP Calculus AB Essential Understandings, Learning Objectives, and Essential Knowledge as published in the *AP Calculus AB and BC Course and Exam Description (Effective Fall 2016).* These items are outlined more specific and by unit in the preceding AP Calculus AB Unit Plan.**

Understanding(s): ***Students will understand . . .***	**Essential Question(s):**
1. Each and every concept covered throughout the AP Calculus AB course. This includes but is not limited to content related to: a. Limits b. Derivatives c. Integrals 2. Ways in which calculus is useful in mathematics 3. Ways in which calculus is useful in real-world settings outside of the classroom	• What skills must I possess as a student to succeed in AP Calculus BC? • What specific areas do I need review in order to move forward? • What calculus skills do I need to have mastered in order to succeed on the AP Calculus BC Exam? • How is what I learned last year going to be connected to what I will learn this year?
Learning Objectives: ***Students will know . . .*** • This particular unit is unique as it is a cumulative review of the entire AP Calculus AB course. All major concepts and ideas covered last year will be revisited and reviewed.	***Students will be able to:*** • This particular unit is a cumulative review of the entire AP Calculus AB course. Each skill the students have been exposed to this last year will be reviewed through practice and assessment.

Dates: September	**Unit Plan: Advanced Integration**
Stage 1 – Desired Results	
College Board AP Calculus AB/BC Curriculum Framework Addressed: **All Essential Understandings, Learning Objectives, and Essential Knowledge cited below are taken from the *AP Calculus AB and AP Calculus BC Course and Exam Description (Effective Fall 2016)*** **While at their most basic level, integrals can be thought of as the antiderivative, they have a number of applications critical to the study of calculus concepts. These applications begin with the approximation of a definite integral in the form of a Riemann sum and continue to the computation of definite integrals using basic methods, u-substitution, and later integral by parts and integration using partial fractions. Similar to derivatives, students must be able to apply their understanding the rectilinear motion applications as well**	

as understand that a definite integral can be used as an accumulation function. Finally, students must be able to conceptualize the relationship between differentiation and integration which is formalized in the Fundamental Theorem of Calculus.	
Understanding(s): ***Students will understand . . .*** 1. The derivative has multiple interpretations and applications including those that involves instantaneous rates of change (EU 2.3) 2. The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies (EU 3.2) 3. The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration (EU 3.3) 4. Antidifferentiation is an underlying concept involved in solving separable differential equations. Solving separable differential equations involves determining a function or relation given its rate of change (EU 3.5)	**Essential Question(s):** ● Why is the study of integrals important? ● How can integrals be used and applied to familiar situations we witness in everyday life? ● In what professions, fields, or actual situations might our applications of integrals in this chapter be useful?
Learning Objectives: ***Students will know . . .*** ● For differential equations, Euler's method provides a procedure for approximating a solution or a point on a solution curve (EK 2.3F2) ● An improper integral is an integral that has one or both limits infinite or has an integrand that is unbounded in the interval of integration (EK 3.2D1) ● Improper integrals can be determind using limits of definite integrals (EK 3.2D2) ● Techniques for finding antiderivatives include… integration by parts and nonrepeating linear partial fractions (EK 3.3B5) ● Antidifferentiation can be used to find specific solutions to differential equations with given initial conditions, including applications to motion along a line, exponential growth and decay, and logistic growth (EK 3.5A1) ● The model for logistic growth that arises from the statement "The rate of change of a quantity is jointly proportional to the size of the quantity and the different between the quantity and the carrying capacity" is *dy/dt = ky(a-y)* (EK 3.5B2)	***Students will be able to:*** ● Estimate solutions to differential equations (LO 2.3F) ● Evaluate an improper integral or show that an improper integral diverges (LO 3.2D) ● Calculate antiderivatives (LO 3.3B(a)) ● Evaluate definite integrals (LO 3.3B(b)) ● Analyze differential equations to obtain general and specific solutions (LO 3.5A) ● Interpret, create, and solve differential equations from problems in context (LO 3.5B)

Dates: September/October	Unit Plan: Polar Curves and Parametric and Vector-Valued Functions
Stage 1 – Desired Results	
College Board AP Calculus AB/BC Curriculum Framework Addressed: **All Essential Understandings, Learning Objectives, and Essential Knowledge cited below are taken from the *AP Calculus AB and AP Calculus BC Course and Exam Description (Effective Fall 2016)*** **The applications of polar curves, parametric equations, and vector-valued functions, can be seen across all of the topics studied in AP Calculus AB. These applications include but are not necessarily limited to those regarding limits, derivatives, and integrals. More importantly, the study and understanding of these new curves allow students to work in real-world scenarios based on change along multiple dimensions over time.**	
Understanding(s): ***Students will understand . . .*** 1. The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies(EU 2.1) 2. A function's derivative, which is itself a function, can be used to understand the behavior of a function (EU 2.2) 3. The derivative has multiple interpretations and applications including those that involves instantaneous rates of change (EU 2.3) 4. The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation (EU 3.4) 5. Antidifferentiation is an underlying concept involved in solving separable differential equations. Solving separable differential equations involves determining a function or relation given its rate of change (EU 3.5)	**Essential Question(s):** • In what ways can the concepts of calculus learned previously be applied to polar curves as well parametric and vector-valued functions? • In what professions, fields, or actual situations might our use of polar curves and/or parametric and vector-valued functions in this unit be useful? • How do these relational and functional forms relate to each other as well as functions presented in rectangular form?
Learning Objectives: ***Students will know . . .*** • Methods for calculating derivatives of real-valued functions can be extended to vector-valued functions, parametric functions, and functions in polar coordinates (EK 2.1C7) • For a curve given by a polar equation $r=f(\theta)$, derivatives of r, x, and y with respect to θ and first and second derivatives of y with respect to x can provide information about the curve (EK 2.2A4) • Derivatives can be used to determine velocity, speed, and acceleration for a	***Students will be able to:*** • Calculate derivatives (LO 2.1C) • Use derivatives to analyze properties of a function (LO 2.2A) • Estimate solutions to differential equations (LO 2.3F) • Apply definite integrals to problems involving motion (LO 3.4C) • Apply definite integrals to problems involving area, volume, and length of a curve (LO 3.4D)

particle moving along curves given by parametric or vector-valued functions (EK 2.3C4) ● The definite integral can be used to determine displacement, distance, and position of a particle moving along a curve given by parametric or vector-valued functions (EK 3.4C2) ● Areas of certain regions in the plane can be calculated with definite integrals. Areas bounded by polar curves can be calculated with definite integrals (EK 3.4D1) ● The length of a planar curve defined by a function or by a parametrically defined curve can be calculated using a definite integral (EK 3.4D3)	

Dates: November/December/January/February	**Unit Plan: Sequences and Series**
Stage 1 – Desired Results	
College Board AP Calculus AB/BC Curriculum Framework Addressed: **All Essential Understandings, Learning Objectives, and Essential Knowledge cited below are taken from the *AP Calculus AB and AP Calculus BC Course and Exam Description (Effective Fall 2016)*** **In their final unit of study, AP Calculus BC students are asked to explore the ideas associated with sequences and series. Important concepts include the study of Maclaurin and Taylor series as well as the radius, interval of convergence, and operations on power series. This culminates in the creation of power series that can be used to approximate a function near a specific value with a desired degree of accuracy.**	
Understanding(s): ***Students will understand . . .*** 1. The sum of an infinite number of real numbers may converge (EU 4.1) 2. A function can be represented by an associated power series over the interval of convergence for the power series (EU 4.2)	**Essential Question(s):** ● Why is the study of sequences and series important both in the context of upper-level mathematics but also in regards to the world in which we live? ● How can sequences and series be used and applied to real-world situations? ● In what professions, fields, or actual situations might our use of sequences and series in this chapter be useful?
Learning Objectives: ***Students will know . . .*** ● The nth partial sum is defined as the sum of the first n terms of a sequence (EK 4.1A1) ● An infinite series of numbers converges to a real number S (or has sum S), if and only if the limit of its sequence of partial sums exists and equals S (EK 4.1A2) ● Common seires of number include geometric series, harmonic series, and p-series (EK 4.1A3)	***Students will be able to:*** ● Determine whether a series converges of diverges (LO 4.1A) ● Determine or estimate the sum of a series (LO 4.1B) ● Construct and use Taylor polynomials (LO 4.2A) ● Write a power series representing a given function (LO 4.2B) ● Determine the radius and interval of convergence of a power series (LO 4.2C)

- A series may be absolutely convergent, conditionally convergent, or divergent (EK 4.1A4) - If a series converges absolutely, then it converges (EK 4.1A5) - In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of number converges or diverges are the nth term test, the comparison test, the limit comparison test, the integral test, the ratio test, and the alternating series test (EK 4.1A6) - If a is a real number and r is a real number such that |r|<1, then the geometric series $\sum_{n=0}^{\infty} ar^n = \frac{1}{1-r}$ (EK4.1B1) - If an alternating series converges by the alternating series test, then the alternating series error bound can be used to estimate how close a partial sum is to the value of the infinite series (EK 4.1B2) - If a series converges absolutely, then any series obtained from it by regrouping or rearranging the terms has the same value (EK 4.1B3) - The coefficient of the nth-degree term in a Taylor polynomial centered at x=c for the function *f* is $\frac{f^{(n)}(c)}{n!}$ (EK 4.2A1) - Taylor polynomials for a function *f* centered at x=c can be used to approximate function values of *f* near x=c. (EK 4.2A2) - In many cases, as the degree of a Taylor polynomial increases, the nth-degree polynomial will converge to the original function over some interval (EK 4.2A3) - The Lagrange error bound can be used to bound the error of a Taylor polynomial approximation to a function (EK 4.2A4) - In some situations where the signs of a Taylor polynomial are alternating, the alternating series error bound can be used to bound the error of a Taylor polynomial approximation to the function (EK 4.2A5) - A power seriers is a series of the form $\sum_{n=0}^{\infty} a_n(x-r)^n$ where *n* is a non-negative integer, $\{a_n\}$ is a sequnces of real numbers, and *r* is a real number (EK 4.2B1) - The Maclaurin series for sin(*x*), cos(*x*), and e^x provide the foundation for constructing	

Maclaurin series for other functions (EK 4.2B2) ● The Maclaurin series for 1/(1-x) is a geometric series (EK 4.2B3) ● A Taylor polynomial for f(x) is a partial sum of the Taylor series for f(x) (EK 4.2B4) ● A power series for a given function can be derived by various methods (e.g. algebraic processes, substitutions, using properties of geometric series, and operations on known series such as term-by-term integration or term-by-term differentiation) (EK 4.2B5) ● If a power series converges, it either converges at a single point or has an interval of convergence (EK 4.2C1) ● The ratio test can be used to determine the radius of convergence of a power series (EK 4.2C2) ● If a power series has a positive radius of convergence, then the power series is the Taylor series of the function to which it converges over the open interval (EK 4.2C3) ● The radius of convergence of a power series obtained by term-by-term differentiation or term-by-term integration is the same as the radius of convergence of the original power series (EK 4.2C4)	

Dates: March/April/May	**Unit Plan: Application of Calculus Concepts**
Stage 1 – Desired Results	
College Board AP Calculus AB/BC Curriculum Framework Addressed: **This unit is designed to review all AP Calculus AB & BC Essential Understandings, Learning Objectives, and Essential Knowledge as published in the *AP Calculus AB and BC Course and Exam Description (Effective Fall 2016).* *This unit is designed to review course content for the AP Exam.***	
Understanding(s): ***Students will understand . . .*** 1. Each and every concept covered throughout the AP Calculus AB & BC courses. This includes but is not limited to content related to: a. Limits b. Derivatives c. Integrals d. Sequences and Series 2. Ways in which calculus is useful in mathematics 3. Ways in which calculus is useful in real-world settings outside of the classroom	**Essential Question(s):** ● What concepts and skills do I need to review in order to be fully prepared for the AP Calculus Exam? ● What are my strengths and weaknesses in regard to the content that will be on the AP Calculus Exam? ● How are each of the major areas, specifically limits, derivatives, and integrals related and how will they be presented on the AP Calculus Exam?

Learning Objectives: ***Students will know . . .***	***Students will be able to:***
● This particular unit is unique as it is a cumulative review of the entire AP Calculus AB & BC courses. All major concepts and ideas covered will be revisited and reviewed.	● This particular unit is a cumulative review of the entire AP Calculus AB & BC courses. Each skill the students have been exposed to this last year will be reviewed through practice and assessment.